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Transient Response of a Solid–Liquid Model Biological Fluidised Bed to a Step Change in Fluid Superficial Velocity

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ABSTRACT

The evolution of a solid–liquid model biological fluidised bed under a step change in fluid superficial velocity is described. During a transient step change, the fluidised bed divides into a top zone which remains at the initial porosity and a bottom zone which settles at the final porosity. The interface of discontinuity in porosity moves progressively upwards through the fluidised bed. The velocity at which the top of the fluidised bed expands or contracts and the upward velocity of the porosity transition interface depend only upon the initial and final states of the bed porosity and the fluid superficial velocity. This results in a linear evolution with time of the total bed height and the height of porosity transition interface. The proposed model is well suited to describe the transient response of low-density particles in a fluidised bed, such as encountered in biological systems, to a sudden change of liquid superficial velocity. The model was validated experimentally.

Key words: fluidisation, solid–liquid, biological, unsteady state, dynamic, model.

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NOTATION

A Reactor section (L^2)
H Height (L)
n Expansion index
r Correlation coefficient
u Fluid superficial velocity ($L\ T^{-1}$)
U Velocity (solid phase) ($L\ T^{-1}$)
V Volume (L^3)
ε Bed porosity
ρ Density (M/L^3)
τ Characteristic time of the transient phase (T)

Subscripts
a Porosity transition interface
f Final
o Initial
P Average for the particles
S Solid
t Bed top
tz Top zone
T Terminal settling

1 INTRODUCTION

Solid–liquid fluidised bed systems have found many applications in chemical and, more recently, in biochemical processes because of their good heat and mass transfer properties combined with high bed homogeneity. However, the fundamental understanding of solid–liquid fluidisation is still at an early stage.

The steady state behaviour of a solid–liquid fluidised bed is well documented.[1,2] To develop a dynamic model for (biological) fluidised bed processes requires a non-steady-state equation for the fluidisation. Very few papers deal with the dynamics of solid–liquid fluidised bed reactors.[3,4]

Recently, a simple model described the transient response of a solid–liquid fluidised bed to a step change in fluid superficial velocity.[5] The present work develops this model further and validates it experimentally with low-density particles so as to model biological processes in fluidised beds.

1.2 Qualitative analysis

1.2.1 Expansion of a solid–liquid fluidised bed following a step increase in fluid superficial velocity

A sudden increase in the fluid superficial velocity from u_o to u_f induces the expansion of the solid–liquid fluidised bed as a consequence of two phenomena (Fig. 1). First, the bed moves upwards in order to obey the generalised law of Richardson and Zaki; see eqn (4), Fig. 1(a2). Secondly, the upward motion of the fluidised bed clears a

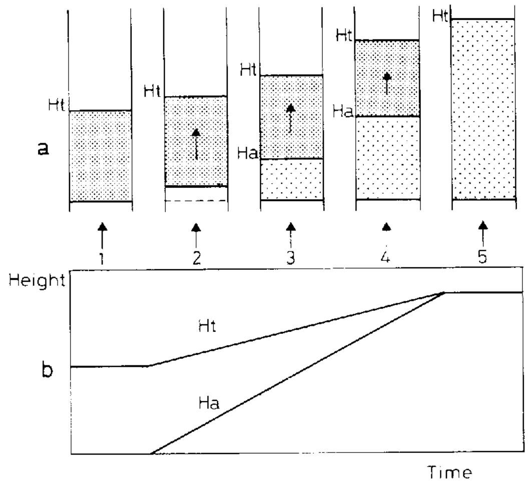

Fig. 1. (a): Evolution of a fluidised bed during an expansion step following a step increase in fluid superficial velocity, u. (b): Evolution of the total bed height, H_t, and the height of the porosity transition interface, H_a, as a function of time.

zone free of solid particles beneath the bed; see Fig. 1(a2). Particles falling from the base of the bed create an expanded bed at the new porosity (or liquid volume fraction) ε_f, related to the new fluid superficial velocity, u_f, according to Richardson and Zaki's law; see eqn (1) Fig. 1 (a3 and a4). Therefore, two zones with two different porosities can be observed during the transient phase: first, a top zone with the initial porosity, ε_o, in an upward movement, the height (H_t–H_a) of which decreases with time; secondly, a bottom zone with the final porosity, ε_f, without ascensional movement, the height of which increases with time. The two zones are separated by an interface, at the height H_a, where the porosity transition takes place. This interface moves progressively upwards. At the end of the transient state, the interface disappears: H_a becomes equal to H_t; see Fig. 1(a5).

1.2.2 Contraction of a solid–liquid fluidised bed following a step decrease in the fluid superficial velocity

A sudden decrease in fluid superficial velocity, from u_o to u_f, induces the contraction of the solid–liquid fluidised bed as a consequence of two phenomena (Fig. 2). First, the bed moves downwards to obey the generalised law of Richardson and Zaki; see eqn (4) Fig. 2(2a). Secondly, the downwards motion of the fluidised bed tends to promote a packing at the base of the reactor. However, the resulting porosity, ε_f, does not decrease below the value predicted by Richardson and Zaki's law— see eqn (1)—for the new fluid superficial velocity, u_f; see Fig. 2(a3). The fluidised bed divides into two zones, as in the case of fluidised bed expansion, but in the contraction step, the top zone moves downwards and has the lowest porosity; see Fig. 2 (a3 and a4). The interface moves progressively upwards and disappears at the end of the transient phase: H_t becomes equal to H_a; see Fig. 2(a5).

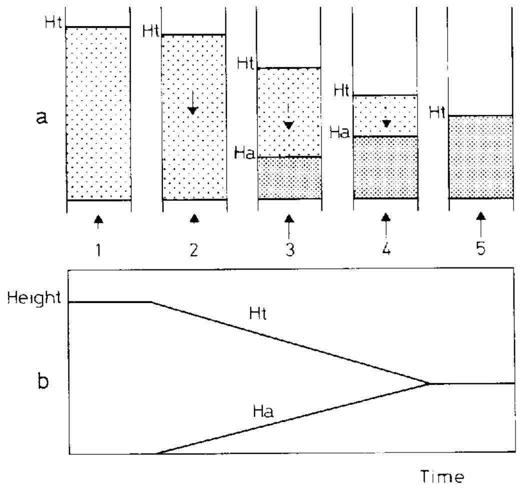

Fig. 2. (a): Evolution of a fluidised bed during a contraction step following a step decrease in fluid superficial velocity, u. (b): Evolution of the total bed height, H_t, and the height of the porosity transition interface, H_a, as a function of time.

1.3 Quantitative analysis

1.3.1 Generalisation of Richardson and Zaki's equation

Richardson and Zaki[2] were the first to observe that, when the fluid superficial velocity, u, is increased in a fluidised bed, the porosity of the fluidised bed increases in turn, resulting in the increase of the settling velocity of the now more dispersed particles. The reverse was observed too. Steady-state conditions in a fluidised bed imply that the net average particle velocity, U_P, is equal to zero. Richardson and Zaki expressed their observation under these steady-state conditions, by the following equation:

$$u = U_T \varepsilon^n \quad (1)$$

Richardson and Zaki's law states that under steady-state conditions the porosity of the fluidised bed, ε, is only a function of the relative velocity between a solid particle and the fluid, namely, between the fluid superficial velocity, u, and the terminal settling velocity of the particles, U_T, that is the terminal settling velocity of an isolated particle in the same fluid.

If the net average particle velocity, U_P, is equal to zero under steady-state conditions, then Richardson and Zaki's law also states that, if u is the fluid superficial velocity, $U_T\varepsilon^n$ must have a value equal to the relative particle velocity in the moving fluid. In other words, $U_T\varepsilon^n$ expresses that the velocity of particles in a given fluid is the fraction ε^n of the terminal settling velocity U_T, a fraction thus depending on the porosity. This is another way to express Kynch's theory that the settling velocity of solid particles in a fluid decreases in a nonlinear fashion with increasing concentration of the particles in the fluid.

Dividing both sides of eqn (1) by the porosity, ε, yields:

$$u/\varepsilon = U_T \varepsilon^{n-1} \quad (2)$$

u/ε represents the true superficial velocity in the fluidised bed whereas u represents the fluid superficial velocity in the absence of solid particles.

Under dynamical conditions, the average particle velocity, U_p, is no longer equal to zero. To comply with Richardson and Zaki's law, eqn (2) must then be rewritten:

$$u/\varepsilon - U_P = U_T \varepsilon^{n-1} \quad (3)$$

where $u/\varepsilon - U_P$ is the net fluid superficial velocity relative to the particles now in net movement.

Multiplying both sides of eqn (3) by the porosity ε yields:

$$u - U_P \varepsilon = U_T \varepsilon^n \quad (4)$$

Equation (4) is the generalisation of Richardson and Zaki's law for a solid–liquid fluidised bed in a transient or dynamical state.

1.3.2 Dynamics of a fluidised bed under a step change in fluid superficial velocity
During the transition step—see Fig. 1(a1–a5), Fig. 2(a1–a5)—the balance around the solid volume in the fluidised bed can be written as follows:

(Solid volume in bottom zone) + (Solid volume in top zone)
= (Total solid volume)

$$(1-\varepsilon_f)AH_a + (1-\varepsilon_o)A(H_t - H_a) = V_s \quad (5)$$

where ε_o and ε_f are, respectively, the initial and final bed porosities (or liquid volume fractions), A is the cross-sectional area of the reactor, and H_t and H_a, respectively, are the total bed height and the height of the porosity transition interface.

Equation (5) can be rewritten in the following form:

$$(\varepsilon_o - \varepsilon_f)AH_a + (1-\varepsilon_o)AH_t = V_s \quad (6)$$

Dividing both sides of eqn (6) by the reactor section, A, and taking the derivative relative to time, t, yields:

$$(\varepsilon_o - \varepsilon_f)dH_a/dt + (1-\varepsilon_o)dH_t/dt = 0 \quad (7)$$

Note that:

$$dH_a/dt = U_a \quad (8)$$

and

$$dH_t/dt = U_t \quad (9)$$

where U_t and U_a are the velocities, respectively, of the top of the bed and of the porosity transition interface. Combining eqns (7), (8) and (9) yields:

$$(\varepsilon_f - \varepsilon_o)U_a = (1-\varepsilon_o)U_t \quad (10)$$

During the time interval t to $t+dt$, the porosity transition interface moves a height interval dH_a which defines thereby a volume A dH_a. The liquid volume balance around this volume $A\,dH_a$ during the time interval dt can be written:

$$\begin{pmatrix}\text{Input liquid volume}\\ \text{between } t \text{ and } t+dt\end{pmatrix} - \begin{pmatrix}\text{Output liquid volume}\\ \text{between } t \text{ and } t+dt\end{pmatrix}$$

$$= \begin{pmatrix}\text{Liquid volume}\\ \text{at } t=t+dt\end{pmatrix} - \begin{pmatrix}\text{Liquid volume}\\ \text{at } t=t\end{pmatrix}$$

$$u_f A dt - u_{tz} A dt = \varepsilon_f A dH_a - \varepsilon_o A dH_a \qquad (11)$$

where u_{tz} is the fluid superficial velocity in the top zone of the fluidised bed. Dividing eqn (11) by the reactor section, A, and the time interval, dt, and combining with eqn (8), yields:

$$u_f - u_{tz} = (\varepsilon_f - \varepsilon_o)U_a \qquad (12)$$

Combining eqns (10) and (12) yields:

$$(1-\varepsilon_o)U_t = u_f - u_{tz} \qquad (13)$$

Before changing the fluid superficial velocity from u_o to u_f, the fluidised bed was in steady-state conditions and the average particle velocity, U_P, was equal to zero. Hence, one can apply eqn (1):

$$u_o = U_T \varepsilon_o^n \qquad (1)$$

During the transition step, first the upper zone moves as a whole. Secondly, the average particle velocity, U_P, remains no longer equal to zero but becomes equal to the bed top velocity, U_t. Thirdly, the porosity remains however at its initial value, ε_o. Equation (4) applies and can be written:

$$u_{tz} - U_t \varepsilon_o = U_T \varepsilon_o^n \qquad (14)$$

Combining eqns (1) and (14) yields:

$$u_{tz} \varepsilon_o U_t = u_o \qquad (15)$$

Combining eqns (13) and (15) yields:

$$U_t = u_f - u_o \qquad (16)$$

This equation gives the value of the bed top velocity, U_t, from parameters easy to measure. Equation (16) states that the bed top velocity, U_t, only depends on the initial and final state. Consequently, it remains constant during the transient step.

The velocity of the porosity transition interface, U_a, can be deduced from eqn (10). Equation (10) states that the velocity of the porosity transition interface, U_a, also remains constant during the transient step, since it only depends on the parameter values of the initial and final state.

Integration of eqns (8) and (9) gives the value of the height of the porosity transition interface, H_a and the total bed height, H_t, as a function of time, t, during the transient step:

$$H_a = H_{ao} + U_a t \qquad (17)$$

$$H_t = H_{to} + U_t t \qquad (18)$$

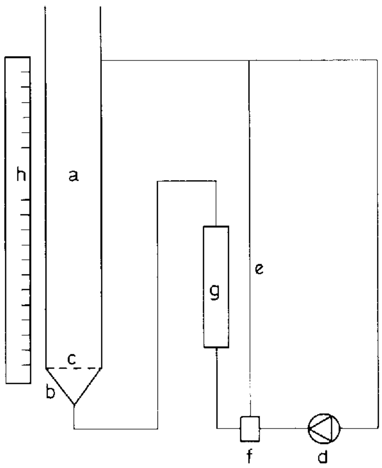

Fig. 3. Experimental set up. (a): Column to contain the fluidised bed; (b): cone; (c): grid; (d): pump, (e): short circuit; (f): watergate; (g): rotameter; (h) graduated rule.

where H_{ao} and H_{to} are the initial heights, respectively, of the porosity transition interface and of the total bed. The theoretical value of the initial height of the porosity transition interface is, as a matter of fact, equal to zero.

2 MATERIALS AND METHODS

The experimental set-up is depicted in Fig. 3. The cylindric column (a) to contain the fluidised bed was constructed of plexiglass, had an internal diameter of 10·7 cm and an internal volume of 12 dm^3. Its base consisted of a cone (b) topped by a grid (c). The grid was necessary to maintain the solid particles inside the column and to allow for a food fluid distribution at the bottom of the vessel. Water (18°C) was fed to the column by means of a rotative pump (d) with screw (SB-15 Moineau, Atelier Gardier, Belgium). The distribution of liquid flow between the vessel (a) and the short circuit (e) was controlled and measured by a watergate (f) and a rotameter (f). A graduated rule (h) was used for the measurement of the heights of the porosity transition interface and of the total bed.

The solid consisted of a mixture of polyethylene particles and talcum powder. Their density was 1·03 kg dm^{-3} and the mean particle diameter was 3·3 mm. The volume of solid in the column, V_s, was 1·9 dm^3.

3 RESULTS

3.1 Characterization of the parameters of Richardson and Zaki's law

For different fluid superficial velocities, u, under steady-state conditions, the porosity, ε, can be computed by:

$$\varepsilon = 1 - V_s/V_{Bed} = 1 - V_s/AH_t \qquad (19)$$

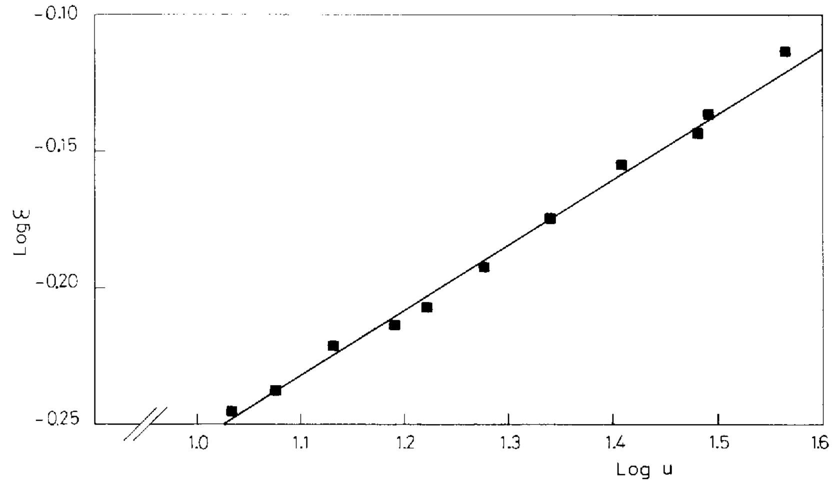

Fig. 4. Evolution of the porosity, ε, at increasing fluid superficial velocities, u, under steady-state conditions.

where V_s and V_{Bed} are the solid and total volume of the fluidised bed.

The log–log plot of bed porosity, ε, versus the liquid superficial velocity, u, for different steady-state conditions of fluidisation yields a linear distribution of date (Fig. 4). The terminal settling velocity of particles, U_T ($112\ \mathrm{m\ h^{-1}}$), and the expansion index, n (4·15), were estimated by regression of the data according to eqn (1):

$$u = 112\varepsilon^{4\cdot 15} \tag{20}$$

The terminal settling velocity, U_T, can be also determined experimentally as the free falling velocity in a water column. This yields the value of $98 \pm 10\ \mathrm{m\ h^{-1}}$, which agrees reasonably with the prediction of Richardson and Zaki's law.

3.2 Evolution of the total bed height, H_t with time

Experiments were carried out to follow the change in the total bed height, H_t, with time, t, following a step change in fluid superficial velocity, u. Figure 5 presents an example of an expansion step following a step increase in fluid superficial velocity, u, which resulted in a linear distribution of the data. Consequently, a linear regression of the total bed height, H_t, with time, t, according to eqn (18), was evaluated for each experiment and allowed the calculation of the velocities of the bed top, U_t, and the initial heights of the bed top, H_{t0}. The bed top velocities, U_t, were also calculated with the help of eqn (16). The results are summarized in Table 1. The experimental uncertainty on the bed top height was approximately 1 cm. The mean deviation between the calculated values and the experimental values for the velocities of the bed top, U_t, was estimated at 3%. The experimental uncertainty for the latter parameter was also estimated at 3%.

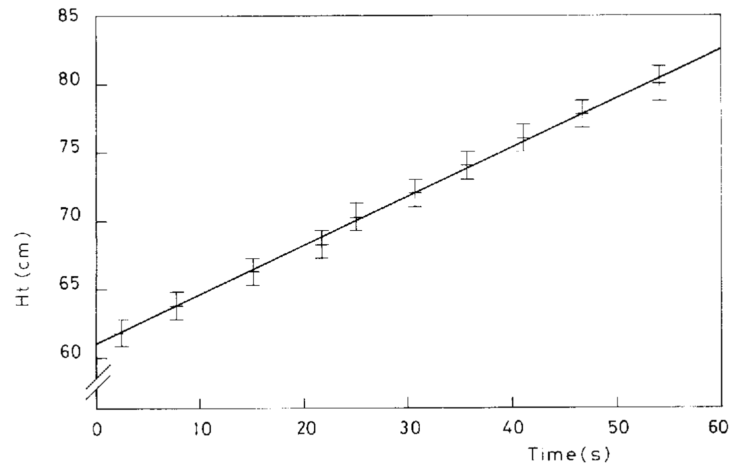

Fig. 5. Evolution with time of the total bed height, H_t, during an expansion step following a step increase in the fluid superficial velocity from $u_0 = 10{\cdot}6$ m h^{-1} to $u_f = 23{\cdot}5$ m h^{-1}. H_t (cm) = (61 + 13·1 (m h^{-1}) 1/36 t (s)) cm; $r = 0{\cdot}999$.

3.3 Evolution of the height of the porosity transition interface

Experiments were performed to follow the change in the height of the porosity transition interface, H_a, with time, t, following a step change in fluid superficial velocity, u. Figure 6 shows an example of a contraction step following a step decrease in fluid superficial velocity. The data distribution appears linear. Consequently, the linear regression of the height of the porosity transition interface, H_a, with time, t, according to eqn (17), was realised for each experiment, which allowed the determination of the initial height of the porosity transition interface, H_{ao}, and of the velocity, U_a, at which the porosity transition interface moves upwards. The porosity transition interface velocity, U_a, was also calculated with the help of eqn (10). The porosity values for the initial and final states were computed using eqn (20). These results are presented in Table 2. The porosity transition interface appears, in fact, as a zone of a few centimetres of height, especially in the expansion steps. This introduces an uncertainty in the experimental determination of the height of the porosity transition interface, H_a, of approximately 1·5 cm for the contraction steps and 3 cm for the expansion steps. The mean deviation between the calculated values and the experimental values for the velocity, U_a, of the porosity transition interface was estimated at 7% whereas the experimental uncertainty was of 4%.

4 DISCUSSION AND CONCLUSION

This paper is intended more specifically at fluidised beds of biological aggregates. Since biological aggregates are difficult to manipulate, and their physical properties are ill-defined and susceptible to changes with time, plastic particles that

TABLE 1
Velocities of the Change in Total Bed Height Following Step Changes in Fluid Superficial Velocity

u_o (*m h*$^{-1}$)	u_f (*m h*$^{-1}$)	H_{to} *exper.*[a] (*cm*)	H_{to} *regres.*[b] (*cm*)	U_t *calc.*[c] (*m h*$^{-1}$)	U_t *regres.*[b] (*m h*$^{-1}$)	r[d]
Expansion						
7·4	11·2	55	55	3·8	3·6 ± 0·2	0·997
12·2	17·4	61	61	5·2	5·4 ± 0·3	0·994
8·0	14·9	49	50	6·9	6·9 ± 0·4	0·996
8·0	16·9	55	56	8·8	9·1 ± 0·3	0·997
10·6	23·5	61	60	12·9	13·1 ± 0·2	0·999
9·5	23·2	60	61	13·7	13·7 ± 0·5	0·999
15·4	30·3	66	66	14·9	15·1 ± 0·4	0·998
17·3	34·1	71	71	16·8	15·7 ± 0·5	0·993
9·4	29·2	53	53	19·8	19·3 ± 0·3	0·992
10·0	37·7	60	59	27·7	25·8 ± 0·2	1·000
10·3	38·5	60	61	28·2	27·9 ± 0·3	0·979
Contraction						
30·3	23·1	97	96	−7·2	−7·4 ± 0·1	1·000
23·1	14·5	80	80	−8·6	−8·8 ± 0·1	1·000
17·1	8·2	70	69	−8·9	−8·2 ± 0·6	0·984
18·3	8·8	70	70	−9·5	−9·2 ± 0·3	0·997
16·5	6·6	68	68	−9·9	−10·3 ± 0·5	0·997
36·9	26·2	115	115	−10·7	−10·9 ± 0·1	1·000
26·2	13·4	85	85	−12·8	−12·8 ± 0·2	0·999
30·3	13·5	92	92	−16·8	−16·5 ± 0·1	0·990
37·1	8·5	115	113	−28·6	−28·7 ± 0·6	0·998

[a] Values obtained by direct observation.
[b] Values obtained by regression of data using eqn (18).
[c] Values calculated by eqn (16).
[d] Correlation coefficient (see [b]).

appropriately simulate the biological aggregates were used. They have a low density (near 1 kg dm^{-3}), a low terminal settling velocity (near 100 m h^{-1}) and present a dispersion of size and forms. The only important divergence between the plastic particles and the biological aggregates concerns their plasticity and their internal porosity. However, this does not affect the validity of the Richardson and Zaki law.[6]

A good correlation was observed between calculated and experimental values of the bed top velocity, U_t, following a step change in fluid superficial velocity, u, as well as the initial total bed height, H_{to}, obtained by direct measurement or by regression. That the bed top velocity, U_t, is constant with time during the dynamic state is demonstrated by the linearity in Fig. 5. Therefore, eqns (16) and (18) correctly describe the evolution of the total bed height, H_t, during an expansion or a contraction step.

The deviation between the calculated values and the experimental values for the velocity of the porosity transition interface, U_a, is larger than its experimental uncertainty. There are three potential reasons for this. First, the computation of the

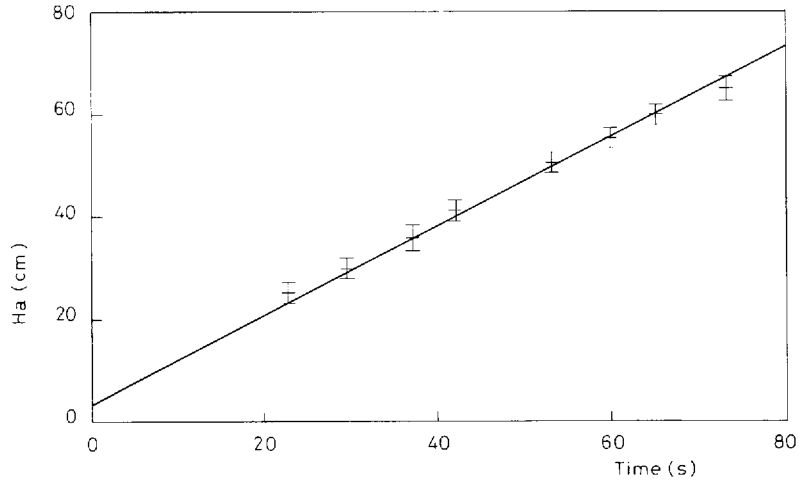

Fig. 6. Evolution with time of the height of the porosity transition interface during a contraction step following a step decrease of the fluid superficial velocity from $u_0 = 27{\cdot}7$ m h^{-1} to $u_f = 13{\cdot}1$ m h^{-1}. H_a (cm) = (3 + 33 (m h^{-1}) 1/36 t (s)) (cm); $r = 0{\cdot}997$.

TABLE 2
Velocities of the Change in Height of the Porosity Transition Interface Following Step Changes in Fluid Superficial Velocity

u_o (*m h*$^{-1}$)	u_f (*m h*$^{-1}$)	H_{ao}[a] (*cm*)	U_a[a] (*m h*$^{-1}$)	U_a[b] (*m h*$^{-1}$)	r[c]
Expansion					
9·5	25·8	8±2	45	41±2	0·998
9·5	25·9	7±1	48	45±1	0·997
10·5	26·5	7±2	51	50±2	0·993
12·0	31·2	2±2	53	54±2	0·998
Contraction					
21·9	9·9	5±2	33	35±2	0·989
25·4	10·9	3±2	34	33±1	0·994
17·7	9·9	6±4	34	37±4	0·969
27·7	13·1	3±1	35	33±1	0·997
27·7	11·7	3±1	35	33±1	0·996
25·4	12·6	3±2	36	33±1	0·996
25·3	13·5	3±1	36	31±1	0·997
31·5	21·8	1±2	40	35±1	0·998

[a] Values obtained by regression of data on eqn (17).
[b] Values calculated by eqn (10) with porosities estimated by eqn (20).
[c] Correlation coefficient (see [a]).

porosity transition interface velocity, U_a, by eqn (10) introduces a non-negligible uncertainty. An error of 1% on the measurement of the fluid superficial velocity can lead to an error up to 4% for the calculated value of the porosity transition interface velocity, U_a. Secondly, the porosity transition interface has a defined broadness which introduces an uncertainty on the height of the porosity transition interface, H_a, of 1·5–3 cm and can lead to an error of 3% on the experimental value of the velocity of the porosity transition interface, U_a. Thirdly, the porosity transition interface was difficult to observe accurately at the beginning of the transient phase due to a non-uniform flow at the inlet to the reactor. The linear regression could only be realised on a part of the transient time interval (in Fig. 6). The latter two reasons also explain the non-nought values of the height of the initial porosity transition interface, H_{ao}.

Nevertheless, examination of Fig. 6 supports the linearity of the increase in height of the porosity transition interface, H_a, with time. Consequently, eqns (10), (16) and (17) give a satisfactory description of the rate of change in the height of the porosity transition interface, H_a, during a transient phase.

The preceeding description does not take into consideration the initial acceleration and final deceleration steps. Gibilaro *et al.*[5] demonstrated that the initial acceleration time interval is very small and, hence, that the initial acceleration step can be neglected. Our observations pointed out that the final deceleration time interval also appears very small and, hence, that the deceleration step can be neglected. Indeed, in several of the expansion tests, the total bed height, H_t, overshot its equilibrium value and returned to its equilibrium position. A simple explanation of the latter phenomenon could be that the velocity of the bed top, U_t, remains constant until the very end of the transient step.

Turbulence tends to deform the bed top and the porosity transition interface in small thin zones. The deformation increases with increasing turbulence. However, the turbulence effect alone does not suffice to explain why the porosity transition zone appears experimentally larger during an expansion step than during a contraction step. This is probably best explained by the observations of Slis *et al.*[3] which are summarised in Fig. 7. Figure 7(a) presents a possible evolution of the porosity, ε, as a function of height, H, during an expansion step. Similarly, Fig. 7(b) represents a possible evolution of the porosity, ε, as a function of height, H, during a contraction step. To obey Richardson and Zaki's law, the average particle velocity, U_P, must follow the trends illustrated, respectively, in Fig. 7(c) and 7(d). It was observed that an absolute increase in average particle velocity, U_P, corresponded to an increasing size of the transition zone with time whereas an absolute decrease in average particle velocity corresponded to a decreasing size of the porosity transition zone.

Strictly speaking, the model of Gibilaro *et al.*[5] is more correct in the case of a contraction step than in the case of an expansion step, but the divergence increases with increasing bed top velocity, U_t. Hence, in the case of a fluidised bed of low density particles, it can be neglected.

The characteristic time of the transient phase, τ, is given by:

$$\tau = (H_{tf} - H_{to})/U_t = H_{tf}/U_a \tag{21}$$

where H_{tf} is the final total bed height. τ is limited to a few minutes for a pilot or an industrial scale reactor and low-density particles. This is an order of magnitude lower compared to other characteristic times of biological processes in fluidised beds such as the mean hydraulic residence time or the bacterial cell doubling

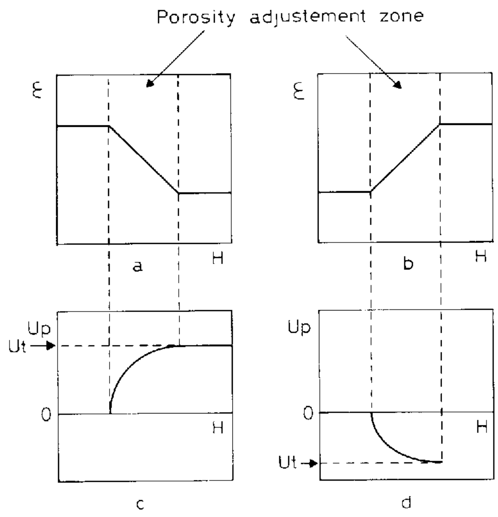

Fig. 7. Characteristics of the porosity transition interface as a function of height, H, during a transient state. (a): Change of the porosity, ε, within the porosity transition interface, during an expansion step, consecutive to a step increase in fluid superficial velocity, u. (b): Change of porosity, ε, within the porosity transition interface, during a contraction step, consecutive to a step decrease in fluid superficial velocity, u. (c): Variation of the main particle velocity, U_P, within the porosity adjustment zone, in the case of an expansion step. (d): Variation of the mean particle velocity, U_P, within the porosity adjustment zone, in the case of a contraction step. (According to Ref. 3.)

time, which are a matter of a few hours, or days, in the case of anaerobic digestion.[7]

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